Hong Kong Mathematics Olympiad (2015/16)

Heat Event (Individual)

香港數學競賽 (2015/16)

初賽項目(個人)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

甲部

Part A

1. 計算 $0.125^{2016}\times\left(2^{2017}\right)^3$ 的值。

   Find the value of $0.125^{2016}\times\left(2^{2017}\right)^3$.

2. 已知方程 $\begin{cases} x_1+x_2=x_2+x_3=x_3+x_4=\cdots=x_{2014}+x_{2015}=x_{2015}+x_{2016}=1 \\ x_1+x_2+x_3+\cdots+x_{2015}+x_{2016}=x_{2016} \end{cases}$，求 x_1 的值。

   Given the equations $\begin{cases} x_1+x_2=x_2+x_3=x_3+x_4=\cdots=x_{2014}+x_{2015}=x_{2015}+x_{2016}=1 \\ x_1+x_2+x_3+\cdots+x_{2015}+x_{2016}=x_{2016} \end{cases}$, find the value of x_1.

3. 有多少個數 x 使得 $\sqrt{2016-\sqrt{x}}$ 為整數？

   How many x are there so that $\sqrt{2016-\sqrt{x}}$ is an integer ?

4. 若 x、y 為整數，有多少對 x、y 且滿足 $(x+1)^2+(y-2)^2=50$。

   If x, y are integers, how many pairs of x, y are there which satisfy the equation $(x+1)^2+(y-2)^2=50$.

5. 63 個連續整數的和是 2016，求緊接該 63 個連續整數後的 63 個連續整數的和。
The sum of 63 consecutive integer is 2016 , find the sum of the next 63 consecutive integers.

6. 已知 8 個整數的平均數、中位數、分佈域及唯一眾數均為 8。若 A 為該 8 個整數中的最大數，求 A 的最大值。
Given that the mean, median, range and the only mode of 8 integers are also 8 . If A is the largest integer among those 8 integers, find the maximum value of A .

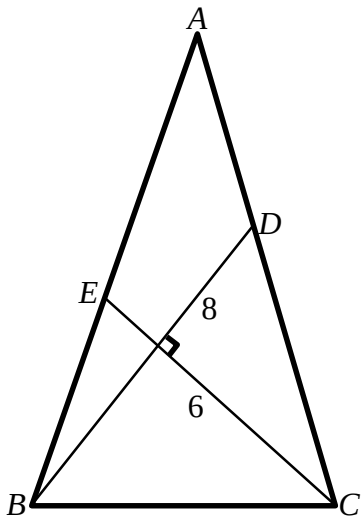
7. 在整數 1 至 500 之間出現了多少個數字「2」？
How many '2's are there in the numbers between 1 to 500 .

8. 某數的 16 進制位數是 1140。而同一數字的 a 進制位數是 240，求 a 。
A number in base 16 is 1140 . The same number in base a is 240 , what is a ?

9. P 點的極坐標為 $(6, 240°)$。若 P 向右平移 16 單位，求 P 的像與極點之間的距離。
The polar coordinates of P are $(6, 240°)$. If P is translated to the right by 16 units, find the distance between its image and the pole.

10. 如圖一，在 ΔABC 中，BD 和 CE 分別是 AC 和 AB 兩邊上的中線，且 $BD \perp CE$。已知 $BD = 8$，$CE = 6$，求 ΔABC 的面積。

As shown in Figure 1, BD and CE are the medians of the sides AC and AB of ΔABC respectively, and $BD \perp CE$. Given that $BD = 8$, $CE = 6$, find the area of ΔABC.



圖一
Figure 1

乙部

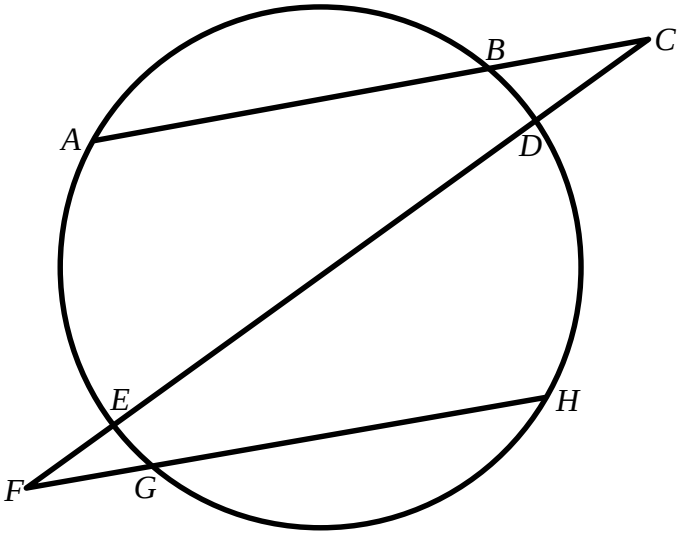
Part B

11. 已知方程 $100[\log(63x)][\log(32x)]+1=0$ 有兩個相異的實數根 α 及 β，求 $\alpha\beta$ 的值。

It is known that the equation $100[\log(63x)][\log(32x)]+1=0$ has two distinct real roots α and β. Find the value of $\alpha\beta$.

12. 如圖二所示，*ABC*，*CDEF* 及 *FGH* 皆為直線，且 *ABC* // *FGH*。 *AB* = 42，*GH* = 40，*EF* = 6 及 *FG* = 8。已知 *ABC* 與 *FGH* 之間的最短距離為 41，求 *BC*。

As shown in Figure 2, *ABC* , *CDEF* and *FGH* are straight lines, *ABC* // *FGH* . *AB* = 42 , *GH* = 40 , *EF* = 6 and *FG* = 8 . Given that the distance between *ABC* and *FGH* is 41 , find *BC*.



圖二

Figure 2

13. 設 *A*、*B* 和 *C* 為三個數字。 利用這三個數字組成的三位數有以下性質：

(a) *ACB* 可以被 3 整除；

(b) *BAC* 可以被 4 整除；

(c) *BCA* 可以被 5 整除；及

(d) *CBA* 的因數數目為單數。

求三位數 *ABC*。

Let *A* , *B* and *C* be three digits. The number formed by these three digits has the following properties:

(a) *ACB* is divisible by 3 ;

(b) *BAC* is divisible by 4 ;

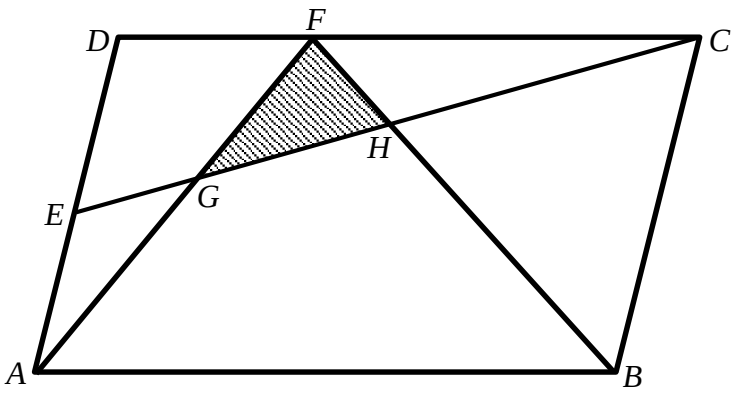
(c) *BCA* is divisible by 5 ; and

(d) *CBA* has an odd number of factors.

Find the 3-digit number *ABC* .

14. 如圖三，在圖中，$ABCD$ 為一平行四邊形。 E 為 AD 的中點及 F 是 DC 上的點且滿足 $DF:FC=1:2$。FA 及 FB 分別相交 EC 於 G 及 H。求 $\frac{ABCD\text{ 的面積}}{\Delta FGH\text{ 的面積}}$ 的值。

As shown in Figure 3 , $ABCD$ is a parallelogram. E is the mid-point of AD and F is a point on DC such that $DF:FC=1:2$. FA and FB intersect EC at G and H respectively. Find the value of $\frac{\text{Area of } ABCD}{\text{Area of } \Delta FGH}$.



圖三

Figure 3

15. 已知數列$\{a_n\}$其中 $a_{n+2}=a_{n+1}-a_n$。若 $a_2=-1$ 及 $a_3=1$，求 a_{2016} 的值。

Given a sequence $\{a_n\}$, where $a_{n+2}=a_{n+1}-a_n$. If $a_2=-1$ and $a_3=1$, find the value of a_{2016} .

完

END